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# Zero-Centric Arithmetic in Communication Theory: A Unified Interpretation of Division-by-Zero Breakdown

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**Abstract**

**Communication systems rely heavily on ratio-based mathematical models such as signal-to-noise ratio (SNR), power gain, impedance relations, and capacity formulas. These expressions assume that denominators remain strictly non-zero so that the computed quantities preserve physical meaning. However, when noise power, input power, current, or bandwidth approach zero, the corresponding expressions become undefined. Traditional interpretations label these cases as producing "infinite SNR," "infinite gain," or "unbounded channel capacity," even though no communication channel or RF system exhibits infinite physical behavior. This paper demonstrates that such contradictions arise not from extreme telecommunication phenomena, but from the collapse of the mathematical operation itself. Using the Zero-Centric Arithmetic framework, expressions of the form $\frac{A}{0}$ found in SNR, gain, and impedance formulations are reinterpreted as non-operations, not as infinite values. This reinterpretation restores coherence between theoretical models and measurable communication behavior, showing that limits such as noise → 0, input → 0, or bandwidth → 0 represent boundaries where classical formulas cease to apply rather than points where physical quantities diverge. The resulting perspective offers a consistent and physically meaningful foundation for analyzing singular behavior across communication systems. The proposed interpretation yields a unified and physically coherent understanding of undefined expressions in communication theory.**

**Keywords: division by zero, non-operation, communication systems, undefined ratios, Zero-Centric Arithmetic, SNR, gain, impedance.**

**الملخص: تعتمد أنظمة الاتصالات اعتمادًا كبيرًا على نماذج رياضية قائمة على النِّسَب، مثل نسبة الإشارة إلى الضوضاء(SNR) ، وكسب القدرة، وعلاقات الممانعة، ومعادلات السعة. تفترض هذه الصيغ أن المقام يبقى غير صفري على نحو صارم، حتى تحتفظ القيم الناتجة بمعناها الفيزيائي. ولكن عندما تقترب قدرة الضوضاء أو قدرة الدخل أو التيار أو عرض النطاق الترددي من الصفر، تصبح هذه التعبيرات غير معرّفة. وتفسّر المقاربات التقليدية هذه الحالات عادةً على أنها تؤدي إلى "SNR لانهائي" أو "Gain لانهائي" أو "سعة قناة غير محدودة"، رغم أن أي قناة اتصال أو نظام RF لا يُظهر سلوكًا فيزيائيًا لانهائيًا في الواقع. تبيّن هذه الورقة أن مثل هذه التناقضات لا تنشأ عن ظواهر اتصالية متطرفة، بل عن انهيار العملية الحسابية نفسها .وباستخدام إطار الحساب الصفري المركزي، تُعاد صياغة التعبيرات من النوع $A/0$التي تظهر في نماذج SNR والكسب والممانعة باعتبارها لا–عمليات وليست قيمًا لانهائية. ويسهم هذا التفسير في استعادة الاتساق بين النماذج النظرية والسلوك القابل للقياس في أنظمة الاتصالات، موضحًا أن الحالات مثل الضوضاء ⟶ 0 أو الدخل ⟶ 0 أو عرض النطاق ⟶ 0 تمثل حدودًا تفقد عندها الصيغ التقليدية صلاحيتها، وليس نقاطًا تتباعد**

  

فيها الكميات الفيزيائية. وتقدّم هذه الرؤية إطارًا موحّدًا ومتّسقًا فيزيائيًا لفهم التعبيرات غير المعرّفة في نظرية الاتصالات وتحليل السلوك الشاذ عبر أنظمة الاتصالات المختلفة.

الكلمات المفتاحية: القسمة على الصفر، عملية غير معرفة، أنظمة الاتصالات، النِّسب غير المعرّفة، الحساب المرتكز على الصفر–Zero) (Centric Arithmetic، نسبة الإشارة إلى الضوضاء(SNR) ، الكسب، الممانعة.

## I. INTRODUCTION

Modern communication systems rely on mathematical relations that describe signal behavior through ratios. Core expressions such as

$$\mathrm{SNR} = \frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}} \tag{1}$$

$$G = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} \tag{2}$$

$$Z = \frac{V}{I} \tag{3}$$

are foundational in analyzing signal quality, gain, impedance, and channel performance. These relationships implicitly assume that denominators noise power, input power, current never reach zero. Under this assumption, the resulting quantities retain their physical meaning.

However, when the denominator approaches zero, each of these expressions enters a region where division is no longer mathematically valid. Traditional communication theory interprets such cases as producing “infinite SNR,” “infinite gain,” or “unbounded impedance.” Yet no communication channel, RF amplifier, or transmission line exhibits infinite behavior in practice. Instead, these expressions become undefined, signaling that the mathematical model has crossed beyond its domain of applicability.

This conceptual mismatch reflects a deeper structural issue: ratio-based models in communication theory are often used outside the range in which division remains a valid operation. Zero-Centric Arithmetic, developed in earlier work [1,2], resolves this inconsistency by interpreting expressions of the form $A/0$ as non-operations, not as extreme electrical or communication quantities. Under this framework, cases such as noise → 0, input → 0, or current → 0 represent points at which the model collapses rather than situations where physical parameters diverge.

By treating these expressions as non-operations, the framework restores coherence between mathematical formulations and the actual behavior of communication systems. It clarifies precisely where classical equations remain valid, where they fail, and why so-called “infinite values” are not real communication phenomena but merely artifacts of algebraic collapse. From this standpoint, the paper presents a unified conceptual framework for understanding division-by-zero breakdowns in communication models. Instead of interpreting undefined ratios as infinite quantities, the Zero-Centric Arithmetic approach demonstrates that they represent **non-operations**, eliminating the contradiction between arithmetic principles and the observable behavior of communication systems.

**Related Work:**

Classical communication texts such as Proakis [3], Haykin [4], and Rappaport [5] introduce ratio-based models that underpin modern communication theory. However, these references address undefined cases only descriptively typically labeling them as "infinite" or "undefined" without offering a structural explanation for the underlying algebraic breakdown. The present work addresses this gap by interpreting such expressions as **non-operations**, providing a coherent operational account that classical sources do not supply.

## II. Theoretical Background

Communication theory relies heavily on mathematical models that express system behavior through ratios. Core relations such as

$$\mathrm{SNR} = \frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}} \tag{4}$$

$$G = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} \tag{5}$$

$$Z = \frac{V}{I} \tag{6}$$

assume that denominators noise power, input power, or current remain strictly non-zero so the resulting quantities preserve physical meaning. However, when these denominators approach zero, the expressions violate the fundamental requirement of division in real-number arithmetic: uniqueness and definability. As discussed in foundational mathematical literature [7], division by zero does not yield a value but collapses the operation itself.

Traditional communication analysis often masks this collapse by describing undefined expressions as "infinite SNR," "infinite gain," or "unbounded impedance." Yet such descriptions do not arise from valid arithmetic and are not physically realizable. For example, (4) becomes undefined at $P_{\mathrm{noise}} = 0$, but standard references describe it as leading to an infinite SNR [3–5]. Similarly, (5) suggests unbounded gain at $P_{\mathrm{in}} = 0$, despite the impossibility of such a condition in any real amplifier or RF chain.

**Definition (Non-operation):**

For any real quantities $A$and $B$, the expression $A/B$is called a **non-operation** if and only if $B = 0$, regardless of the value of $A$. A non-operation denotes the failure of the division process itself; it is **not** a numerical value, not a limit, and not a quantity that can be meaningfully assigned a magnitude. It marks a point at which the arithmetic operation ceases to exist.

This reinterpretation does not alter the axioms of real-number arithmetic; rather, it formalizes the status of expressions already recognized as undefined.

Zero-Centric Arithmetic, introduced in earlier work [1,2], resolves these contradictions by treating expressions of the form $A/0$ as **non-operations**, not as extreme telecommunication quantities. Under this framework, undefined expressions represent boundaries where the mathematical model

ceases to correspond to a realizable communication scenario. A noiseless channel does not produce infinite SNR; the formula simply exits its domain of validity. An input-free amplifier does not generate infinite gain; the ratio becomes non-computable.

This reinterpretation restores coherence between the algebraic forms used in communication theory and the measurable behavior of real-world systems, clarifying where classical equations apply and where they inherently fail.

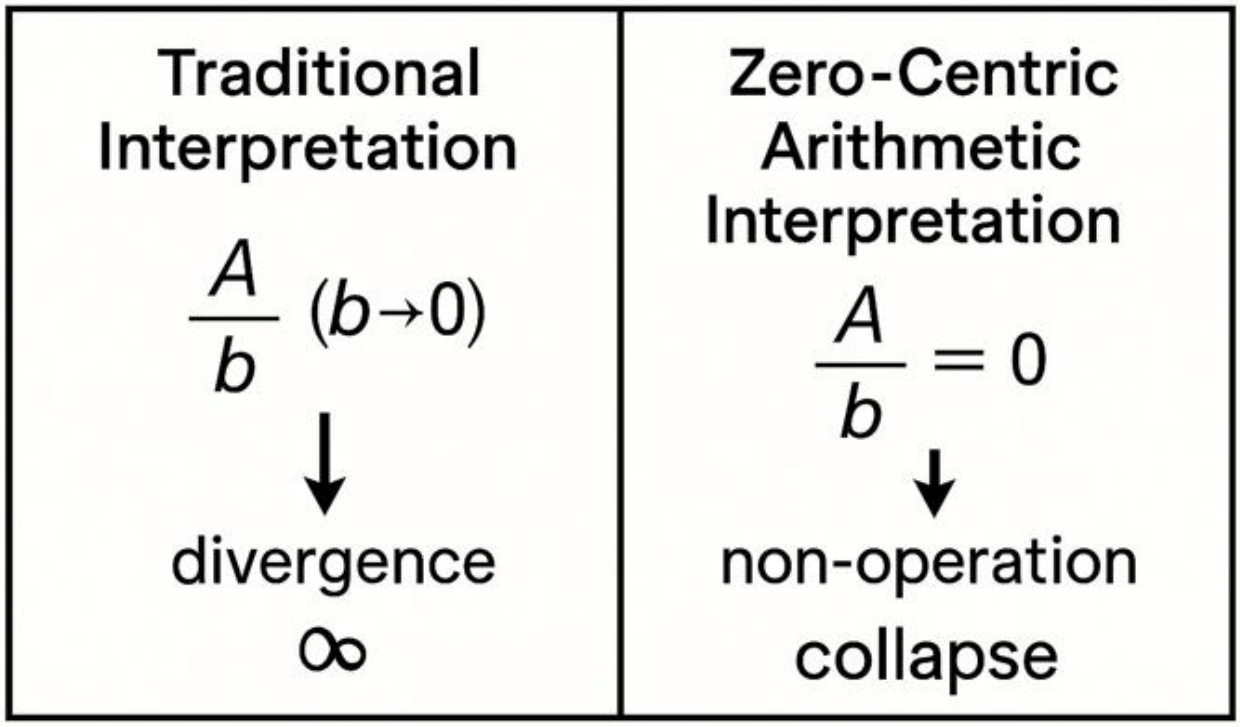


Figure 1. Classical vs. Zero-Centric Interpretation of A/0

Figure 1. Comparison between the traditional interpretation of division-by-zero (left), which treats the expression as diverging to infinity, and the Zero-Centric interpretation (right), which identifies the same expression as a non-operation marking the collapse of the division process.

## III. Fundamental Communication Examples

Division by zero appears naturally in several foundational communication formulas. In each case, the contradiction does not represent extreme communication behavior, but the breakdown of the mathematical model when a denominator becomes zero.

### 3.1 Signal-to-Noise Ratio (SNR) at Zero Noise Power

SNR is defined as

$$\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} \tag{7}$$

When $P_{\text{noise}} = 0$, expression (7) becomes $\frac{P_{\text{signal}}}{0}$, traditionally interpreted as "infinite SNR." Yet no communication channel exhibits infinite clarity or perfect detection. The ratio is simply **undefined**, indicating that the model exits its domain of validity.

Under Zero-Centric interpretation:

$$\frac{P_{\text{signal}}}{0} \rightarrow \text{non-operation}$$

### 3.2 Power Gain at Zero Input Power

Amplifier gain is expressed as

$$G = \frac{P_{\text{out}}}{P_{\text{in}}} \tag{8}$$

When $P_{\text{in}} = 0$, (8) becomes $\frac{P_{\text{out}}}{0}$, and textbooks often describe this as "infinite gain." However, real amplifiers cannot produce output without input power. Thus the ratio becomes non-computable.

The undefined behavior reflects **model collapse**, not infinite amplification.

### 3.3 Impedance in RF Systems at Zero Current

In RF analysis, impedance is defined by

$$Z = \frac{V}{I} \tag{9}$$

At $I = 0$, expression (9) becomes $\frac{V}{0}$, commonly described as "infinite impedance." But an open circuit does not possess infinite magnitude; rather, it is a configuration where the ratio cannot be evaluated.

Thus,

$$Z = \frac{V}{0} \rightarrow \text{non-operation}$$

### 3.4 Channel Capacity at Zero Bandwidth

Shannon capacity is given by

$$C = B\log_2(1 + \text{SNR}) \tag{10}$$

When $B = 0$, the expression does not yield a meaningful capacity value. The model becomes algebraically degenerate, not physically extreme.

Zero bandwidth represents a **boundary where the formula stops being valid**, not a limit where capacity "collapses to zero" in the algebraic sense.

### 3.5 Energy-per-Bit to Noise Ratio $E_b/N_0$ at Vanishing Noise

The relation is

$$\frac{E_b}{N_0} = \frac{P_{\text{bit}}}{N_0} \tag{11}$$

When $N_0 = 0$, the expression becomes undefined. Traditional interpretation labels this as "infinite $E_b/N_0$," but realistic communication systems cannot operate with zero noise density.

The Zero-Centric framework identifies this as a **non-operation**, not a physical state.

### 3.6 Summary of section 3

The examples presented in Section 3 show that the contradictions commonly described in communication theory such as "infinite SNR," "infinite gain," or "unbounded impedance" do not correspond to any physical behavior in real communication systems. Instead, each contradiction emerges from placing zero in the denominator of a ratio-based expression, causing the underlying mathematical operation to collapse. By applying the Zero-Centric Arithmetic framework, these undefined expressions are correctly interpreted as **non-operations**, not as extreme telecommunication quantities. This reinterpretation restores logical consistency between communication formulas and the measurable limits of real channels, clarifying that situations like zero noise, zero input power, or zero current represent points where the classical model fails rather than points where the physical parameters diverge.

Building on this analysis, Table 1 provides a concise comparison between the classical interpretation of these undefined ratios and the Zero-Centric reinterpretation proposed in this paper. While the traditional approach treats expressions such as SNR, gain, impedance, capacity, and $E_b/N_0$ as diverging to infinity when their denominators vanish, the Zero-Centric framework classifies all such cases as **non-operations**. This comparison highlights the shift from viewing these expressions as extreme physical values to recognizing them as points where the mathematical model itself collapses.

Table 1 Conventional Interpretation vs. Zero-Centric Interpretation in Communication Models

| Communication Case | Conventional Interpretation | Zero-Centric Interpretation |
|---|---|---|
| SNR = Psignal / Pnoise with Pnoise = 0 | SNR → ∞ | Undefined ratio → **non-operation** |
| Gain = Pout / Pin with Pin = 0 | Gain → ∞ | Model collapse → **non-operation** |
| Impedance = V / I with I = 0 | Z → ∞ | Open circuit; ratio undefined → **non-operation** |
| Shannon Capacity with B = 0 | Capacity "goes to zero" algebraically | Formula out of domain → **non-operation** |
| Eb/N0 with N0 = 0 | Eb/N0 → ∞ | Not physically realizable → **non-operation** |

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## IV. Discussion

The examples analyzed in Section 3 reveal a unified structural pattern in communication theory: whenever a denominator in a ratio-based expression reaches zero, the resulting contradiction does not indicate an extreme telecommunication condition but signals the collapse of the mathematical model. Traditional treatments describe such cases as "infinite SNR," "infinite gain," or "unbounded impedance," yet none of these outcomes correspond to real channel behavior. Communication systems do not deliver infinite clarity, infinite amplification, or infinite electrical resistance.

The traditional language of "infinite values" is misleading because it suggests the existence of an unbounded physical or mathematical quantity, whereas no such quantity is produced by the model. What actually occurs is an arithmetic collapse masked by terminology that implies physical divergence. Distinguishing between these two interpretations is essential for avoiding conceptual errors in communication theory.

This discrepancy arises because classical models extend algebraic formulas beyond their valid domain. Relations such as

$$\mathrm{SNR} = \frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}, G = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}, Z = \frac{V}{I}$$

all depend on the assumption that denominators never reach zero. When noise power, input power, or current vanish, the arithmetic structure collapses. The undefined forms that emerge are not numerical predictions but indicators that the nominal model no longer describes a physically realizable communication condition.

Zero-Centric Arithmetic provides a consistent reinterpretation of these phenomena by classifying expressions of the form $A/0$ as **non-operations**. Under this framework, "infinite" results are understood not as physical quantities but as artifacts of pushing mathematical expressions beyond their scope. Thus, a noiseless channel does not produce infinite SNR, it marks a point where the SNR formula loses meaning. An amplifier with zero input does not exhibit infinite gain the classical ratio simply becomes undefined. Similarly, impedance becomes non-computable rather than infinite when current is zero.

This reinterpretation resolves long-standing inconsistencies between theoretical expressions and real communication system behavior. It restores conceptual clarity, distinguishes operational limits from algebraic breakdowns, and provides a coherent foundation for understanding singular conditions in communication engineering. By reframing undefined ratios as non-operations, the Zero-Centric perspective strengthens the theoretical integrity of communication models and aligns them with measurable reality.

The framework has implications for teaching, mathematical software validation, and the interpretation of extreme cases in communication system modeling.

**Numerical Stability:**

Simulations often produce overflow, divide-by-zero warnings, or NaN values precisely at the ratio points where Zero-Centric Arithmetic predicts non-operation. This alignment between computational failure and the proposed theoretical framework underscores its practical relevance. It shows that numerical solvers implicitly treat expressions of the form $A/0$ as non-operations rather than divergent quantities, reinforcing the consistency between mathematical theory and computational behavior.

## V. CONCLUSION

This paper has shown that the contradictions commonly attributed to communication models such as infinite SNR, infinite gain, and unbounded impedance do not arise from the physical behavior of communication systems but from the collapse of the mathematical operations underlying the classical formulas. Expressions such as

$$\text{SNR} = \frac{P_{\text{signal}}}{0}, G = \frac{P_{\text{out}}}{0}, Z = \frac{V}{0}$$

are not representations of physical extremes but undefined ratios that mark the limits of validity of traditional communication equations.

Through the Zero-Centric Arithmetic framework, these undefined expressions are reclassified as **non-operations**, clarifying that the apparent infinities in communication theory are artifacts of applying ratio-based models outside their domain of definition. This reinterpretation restores consistency between algebraic formulations and the measurable behavior of real communication systems, distinguishing genuine operational limits from arithmetic breakdowns.

By recognizing that zero noise, zero input power, zero current, and zero bandwidth are not points of physical divergence but boundaries where classical models fail, Zero-Centric Arithmetic provides a coherent and unified foundation for understanding singular behavior in communication engineering. The framework strengthens theoretical clarity and improves the reliability of model interpretation across a wide range of communication scenarios.

Future research may extend the Zero-Centric framework to multi-parameter communication models, matrix-based descriptions of MIMO systems, stochastic channel models, and numerical solvers used in simulation tools. Further work is also needed to investigate whether non-operation boundaries can be used as predictive markers for algorithmic instability or physical system limits.

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